Chapter 12
Kinetic Theory

Q1. A cubic vessel (with face horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of500 ms^{-1} in vertical **direction. The pressure of the gas inside the vessel as observed by us on the ground**

(a) remains the same because 500 ms^{-1} is very much smaller than v_{rms} of the gas.

(b) remains the same because motion **of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.**

(c) will increase by a factor equal to $(v^2_{rms} + (500)^2)/v^2_{rms}$ where v_{rms} was the original mean square velocity of the gas.

(d) will be different on the top wall and bottom wall of the vessel.

Sol: (b) According to the ideal gas law,

P=nRT/V, here temperature of the vessel remain unchanged hence, the
pressure remains same from that point of view.
Now, let us discuss the phenomenon inside the vessel. The gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collisions are perfectly elastic.
The number of collisions per unit volume in a gas remains constant. So, the pressure of the gas inside the vessel remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules with respect to the walls.

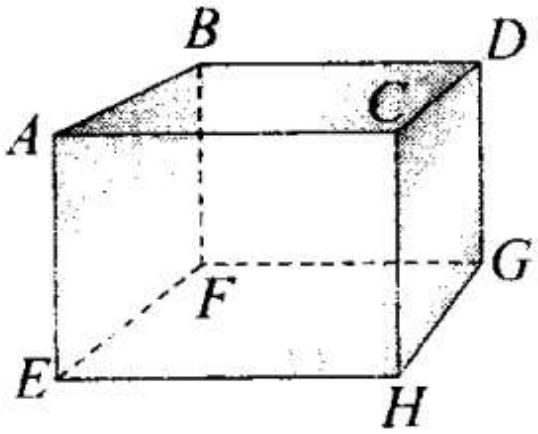
Q2. 1 mole of an ideal gas is contained in a cubical volume V, ABCDEFGH at 300 K (figure). One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time,

(a) the pressure on EFGH would be zero

(b) the pressure on all the faces will be equal

(c) the pressure of EFGH would be double the pressure on ABCD

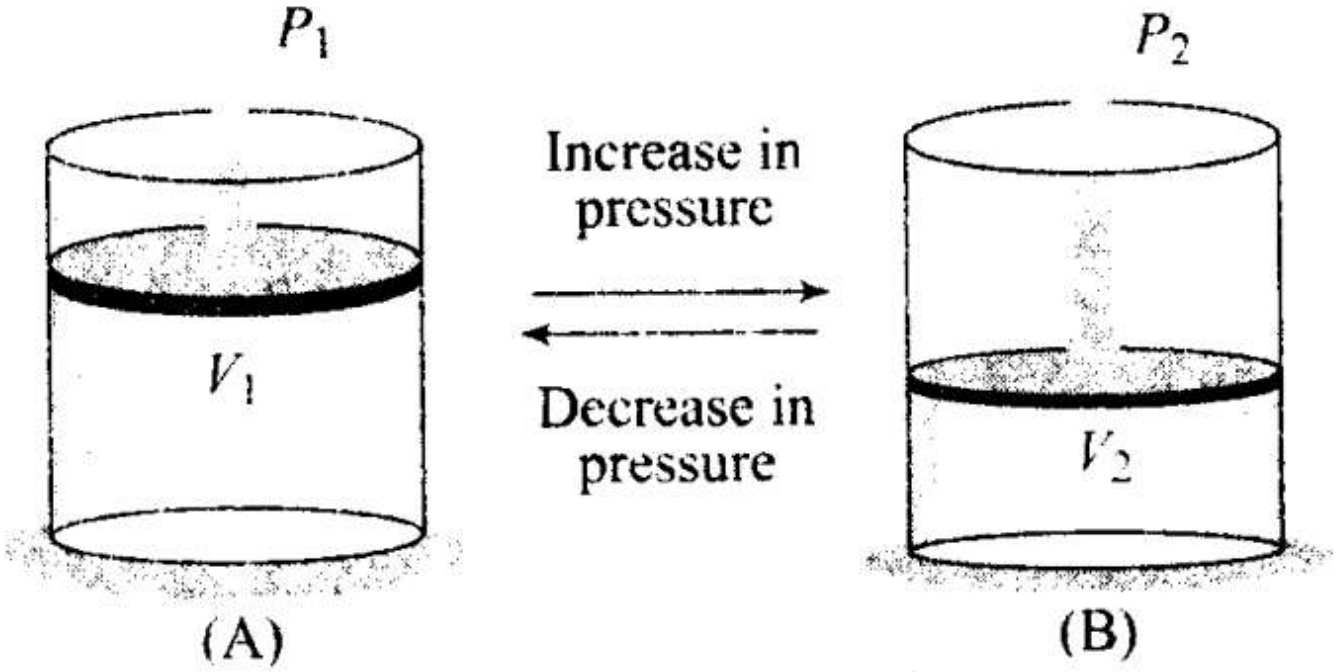
(d) the pressure on EFGH would be half that on ABCD



Sol: (d) In an ideal gas, the gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collisions are perfectly elastic. So, their kinetic energy and momentum remains conserved.
So, the momentum transferred to the face ABCD = 2mv And the gas molecule is absorbed by the face EFGH. Hence it does not rebound. So, momentum transferred to the face EFGH = mv.
And the pressure on the faces is due to the total momentum to the faces. So, pressure on EFGH would be half that on ABCD.

Q3. Boyle's law is applicable for an
(a) adiabatic process
(b) isothermal process
(c) isobaric process
(d) isochoric process
Sol: (b)
Key concept: Boyle's law: For a given mass of an ideal gas at constant temperature, the volume of a gas is

inversely proportional to its pressure.

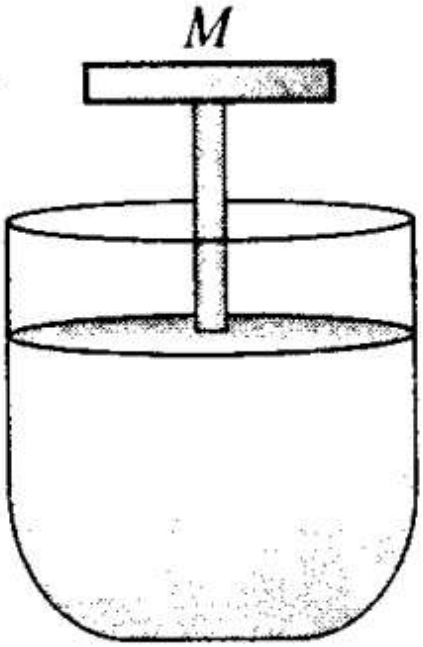


So we can say that when temperature is constant, Boyle's law is applicable.

i.e., PV= nRT= constant
=>PV = constant (at constant temperature)
i.e.. p ∝ 1/V— [where, P = pressure. V= volume]

So, this law is applicable for an isothermal process, in which temperature remain constant.

Q4. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction (figure). If the temperature is increased
(a) both P and V of the gas will change
(b) only P will increase according to Charles' law
(c) V will change but not P
(d) P will change but not V



Sol: (c) The situation is shown in the diagram where an ideal gas is contained in a cylinder, having a piston of mass M.

According to the problem, piston can move up and down without friction. So, the only force present is weight of the piston.

The pressure inside the gas will be

$$P = P_0 + Mg/A = \text{constant}$$

where, P_0 = atmospheric pressure

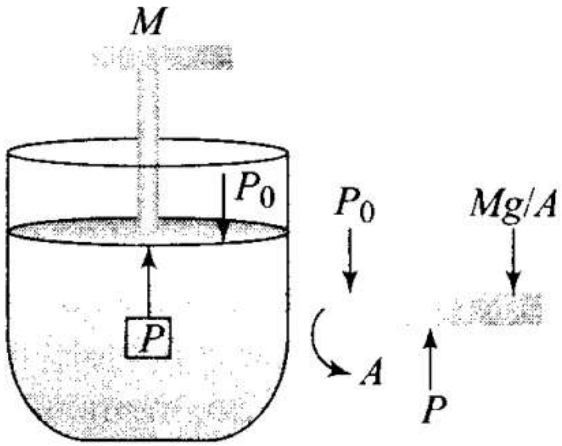
A = area of cross-section of the piston.

Mg = weight of piston

Weight of piston and atmospheric pressure are constant, hence pressure remains constant.

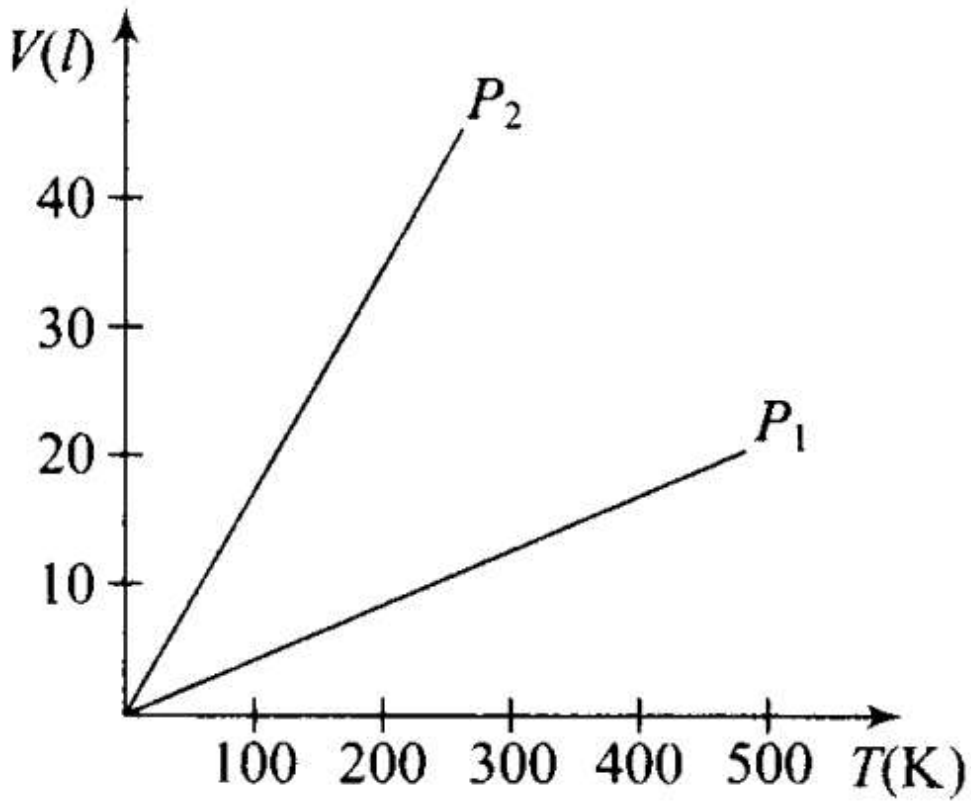
According to ideal gas law,

$$PV = nRT$$



As the pressure remains constant, so if the temperature is increased, only the volume increases as the piston moves up without friction.

Q5. Volume versus temperature graphs for a given mass of an ideal gas are shown in figure. At two different values of constant pressure. What can be inferred about relation between P_1 and P_2 ?



(a)$P_1 > P_2$

(b) $P_1 = P_2$

(c) $P_1 < P_2$

(d) Data is insufficient

Sol. **(a)** We have to consider slope of the V–T graph. From ideal gas equation, the slope of V–T curve is $\frac{V}{T}=\left(\frac{nR}{P}\right)$.

So, if the slope is greater, pressure will be smaller and vice-versa.
We know for an ideal gas,

$$PV = nRT \Rightarrow V = \left(\frac{nR}{P}\right)T$$

Slope of the V–T graph, $m = \frac{dV}{dT} = \frac{nR}{P}$

$$\Rightarrow \quad m \propto \frac{1}{P} \quad [\because nR = \text{constant}]$$

$$\Rightarrow \qquad P \propto \frac{1}{m}$$

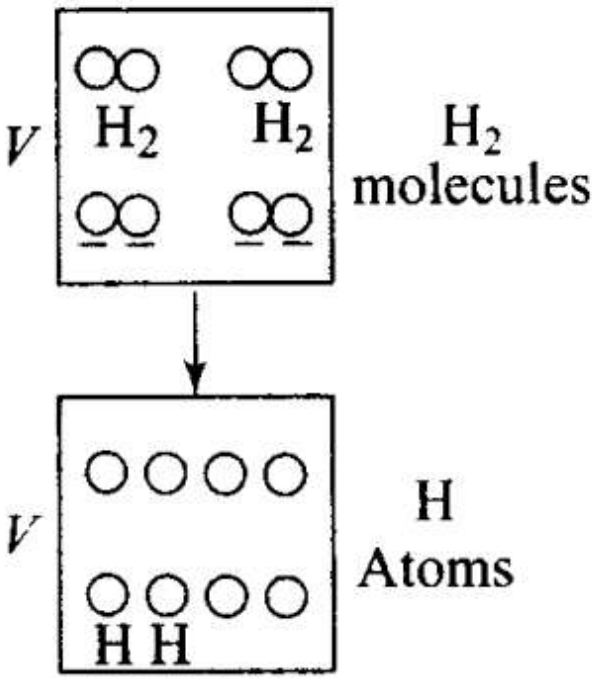
Hence, $\frac{P_1}{P_2} = \frac{m_2}{m_1} < 1$

So, $P_1 > P_2$.
where, m_1 is slope of the graph corresponding to P_1 and similarly m_2 is slope corresponding to P_2.
Hence the correct option is (a), *i.e.* $P_1 > P_2$

Q6. 1 mole of H_2 gas is contained in a box of volume V = 1.00 m^3 at T = 300 K. The gas is heated to a temperature of T= 3000 K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)

(a) same as the pressure initially
(b) 2 times the pressure initially
(c) 10 times the pressure initially
(d) 20 times the pressure initially

Sol: (d) The situation is shown in the diagram, H_2 gas is contained in a box is heated and gets converted to a gas of hydrogen atoms. Then the number of moles would become twice.
According to gas equation,
PV= nRT



P = Pressure of gas, n = Number of moles

R = Gas constant, T = Temperature PV=nRT

As volume (V) of the container is constant. Hence, when temperature (T) becomes 10 times, (from 300 K to 3000 K) pressure (P) also becomes 10 times, as$P \propto T$.

Pressure is due to the bombardment of particles and as gases break, the number of moles becomes twice of initial, so $n_2 = 2n_1$

$$\text{So} \qquad P \propto nT$$

$$\Rightarrow \qquad \frac{P_2}{P_1} = \frac{n_2 T_2}{n_1 T_1} = \frac{(2n_1)(3000)}{n_1(300)} = 20$$

$$\Rightarrow \qquad P_2 = 20P_1$$

Hence, final pressure of the gas would be 20 times the pressure initially.

Q7. A vessel of volume V contains a mixture of 1 mole of hydrogen and 1 mole of oxygen (both considered as ideal). Let $f_1(v)dv$ denotes the fraction of molecules with speed between v and (v + dv**) with $f_2(v)dv$, similarly for oxygen. Then,**

(a) $f_1(v) + f_2(v) = f(v)$ obeys the Maxwell's distribution law

(b) $f_1(v)$, $f_2(v)$ will obey the Maxwell's **distribution law separately**

(c) neither $f_1(v)$nor $f_2(v)$will obey the Maxwell's distribution law

(d) $f_2(v)$ and $f_1(v)$will be the same

Sol: (b)

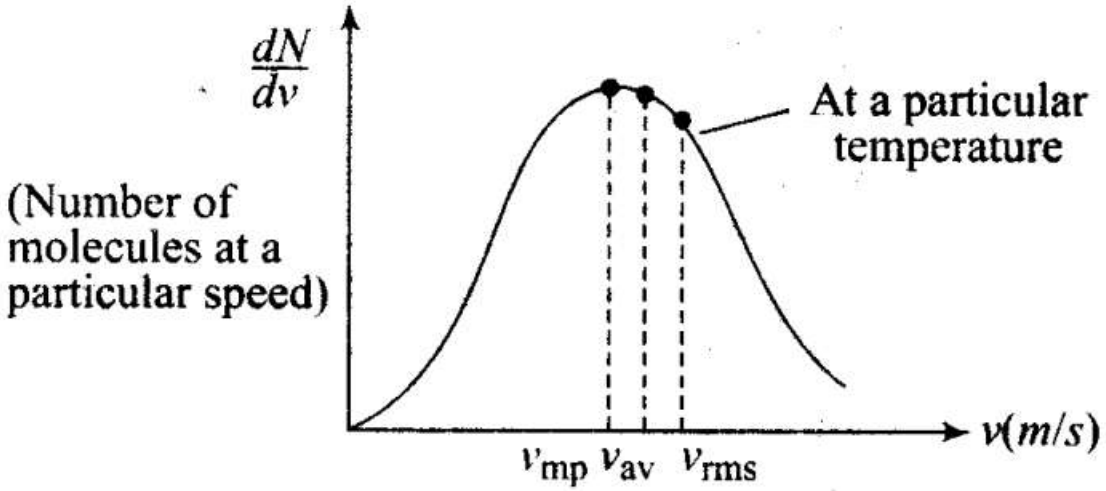
Key concept: Maxwell's Law (or the Distribution of Molecular Speeds):

(1) The v_{rms} gives us a general idea of molecular speeds in a gas at a given temperature. This doesn't mean that the speed of each molecule is v_{rms}. Many of the molecules have speed less than v_{rms} and many have speeds greater than v_{rms}.

(2) Maxwell derived equation gives the distribution of molecules in different speeds as follows:

$$dN = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$$

where dN = Number of molecules with speeds between v and $v + dv$



The masses of hydrogen and oxygen molecules are different.

For a function f(v), the number of molecules dn = f[v), which are having speeds between v and v + dv. The Maxwell-Boltzmann speed distributionfunction (N_v = dn/dv depends on the mass of the gas molecules. For each function $f_1(v)$ and $f_2(v)$, n will be different, hence each function $f_1(v)$ and $f_2(v)$ will obey the Maxwell's distribution law separately.

Q8. An inflated rubber balloon contains one mole of an ideal gas, has a pressure P. volume V and temperature T. If the temperature rises to 1.1 T, and the volume is increased to 1.05 V, the final pressure will be

(a) 1.1 P

(b) P

(c) less than P

(d) between P and 1.1

Sol:(d) According to the equation of ideal gas, PV= nRT

P = pressure

V = volume

n = number of moles of gases

R = gas constant

T = temperature

Thus we have to rewrite this equation in such a way that no. of moles is given by,

$$n = \frac{PV}{RT}$$

As number of moles of the gas remains fixed, hence, we can write

$$\frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2}$$

$$\Rightarrow \quad P_2 = (P_1 V_1)\left(\frac{T_2}{V_2T_1}\right)$$

$$= \frac{(P)(V)(1.1T)}{(1.05)V(T)} \qquad \left[\begin{array}{l} P_1 = P \\ V_2 = 1.05\,V \text{ and } T_2 = 1.1T \end{array}\right]$$

$$= P \times \left(\frac{1.1}{1.05}\right)$$

$$= P(1.0476) \approx 1.05P$$

Hence, final pressure P_2 lies between P and $1.1P$.

More Than One Correct Answer Type

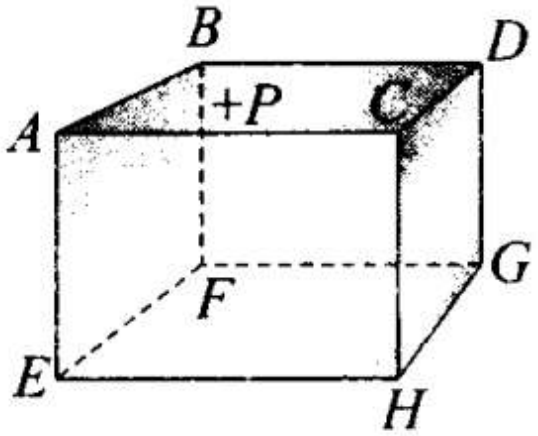
Q9. ABCDEFGH is a hollow cube made of an insulator (figure) face A BCD has positive charge on it. Inside the cube, we have ionised hydrogen.

The usual kinetic theory expression for pressure

(a) will be valid

(b) will not be valid, since the ions would experience forces other than due to collisions with the walls

(c) will not be valid, since collisions with walls would not be elastic
(d) will not be valid because isotropy is lost



Sol: (b, d) According to the problem, ionized hydrogen is present inside the cube, they are having charge. Now, due to the presence of positive charge on the surface A BCD hydrogen ions would experience forces other than the forces due to collision with the walls of container. So, these forces must be of electrostatic nature. Hence, Isotropy of system is lost at only one face ABCD because of the presence of external positive charge. The usual expression for pressure on the basis of kinetic theory will be valid.

Q10. Diatomic molecules like hydrogen have energies due to both translational as well as rotational motion. From the equation in kinetic theory PV = 2/3 E,E is
(a) the total energy per unit volume
(b) only the translational part of energy because rotational energy is very small compared to the translational energy
(c) only the translational part of the energy because during collisions with the wall pressure relates to change in linear momentum
(d) the translational part of the energy because rotational energies of molecules can be of either sign and its average over all the molecules is zero

Sol: (c) According to kinetic theory equation, PV = 2/3 E [where P= Pressure V = volume]
E is representing only translational part of energy. Internal energy contains all types of energies like translational, rotational, vibrational etc. But the molecules of an ideal gas is treated as point masses in kinetic theory, so its kinetic energy is only due to translational motion. Point mass does not have rotational or vibrational motion. Here, we assumed that the walls only exert perpendicular forces on molecules. They do not exert any parallel force, hence there will not be any type of rotation present. The wall produces only change in translational motion.

Q11. In a diatomic molecule, the rotational energy at a given temperature
(a) obeys Maxwell's distribution

(b) have the same value for all molecules

(c) equals the translational kinetic energy for each molecule

(d) is (2/3)rd the translational kinetic energy for each molecule

Sol: (a, d)

Key concept: Kinetic Energy of Ideal Gas:

Molecules of ideal gases possess only translational motion. So they possess only translational kinetic energy.

(i) Kinetic energy of a gas molecule

$$(E_{\text{molecule}}) = \frac{1}{2}mv_{\text{rms}}^2 = \frac{1}{2}m\left(\frac{3kT}{m}\right) = \frac{3}{2}kT \qquad \left[\text{As } v_{\text{rms}} = \sqrt{\frac{3kT}{m}}\right]$$

(ii) Kinetic energy of 1 mole (M gram) gas

$$(E_{\text{mole}}) = \frac{1}{2}Mv_{\text{rms}}^2 = \frac{1}{2}M\frac{3RT}{M} = \frac{3}{2}RT \qquad \left[\text{As } v_{\text{rms}} = \sqrt{\frac{3RT}{M}}\right]$$

(iii) Kinetic energy of 1 gm gas

$$(E_{\text{gram}}) = \frac{3}{2}\frac{R}{M}T = \frac{3}{2}\frac{kN_A}{mN_A}T = \frac{3}{2}\frac{k}{m}T = \frac{3}{2}rT$$

Here m = Mass of each molecule, M = Molecular weight of gas and N_A = Avogadro number = 6.023×10^{23}

According to the problem we have to find the rotational energy of a diatomic molecule in the terms of translation kinetic energy.

First let us check the options by picking them one by one.

(a) Translational kinetic energy and rotational kinetic energy both obey Maxwell's distribution independent of each other.

(b) Rotational kinetic energy is different for different molecule.

(c) Molecules of ideal gases possess only translational motion. So they possess only translational kinetic energy. But in case of non-ideal gas there is a smaller rotational energy.

(d) Here, 2 rotational and 3 translational energies are associated with each molecule. Translation kinetic

energy of each molecule,

$$K_T = \frac{3}{2}kT, \text{ where } k \text{ is constant}$$

Rotational kinetic energy, $K_R = 2\left(\frac{1}{2}kT\right)$

On dividing these two equations, we get

$$\frac{K_R}{K_T} = \frac{(kT)}{\frac{3}{2}kT}$$

$$\Rightarrow \qquad K_R = \frac{2}{3}K_T$$

$\therefore$ Rotational energy at a given temperature is $\left(\frac{2}{3}\right)$rd of translational KE of each molecule.

Important points: Kinetic energy per molecule of a gas does not depend upon the mass of the molecule but only depends upon the temperature of the gas.

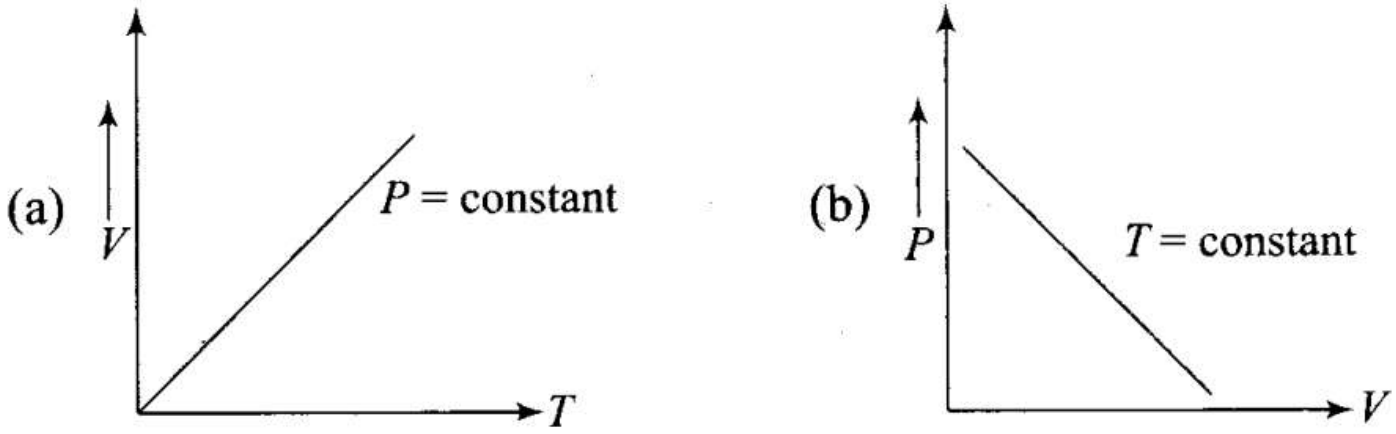
. As $E = \frac{3}{2}kT$ or $E \propto T$, i.e. molecules of different gases say He, H_2 and O_2 etc. at same temperature will have same translational kinetic energy though their rms speed are different. $\left[v_{rms} = \sqrt{\frac{3kT}{m}}\right]$

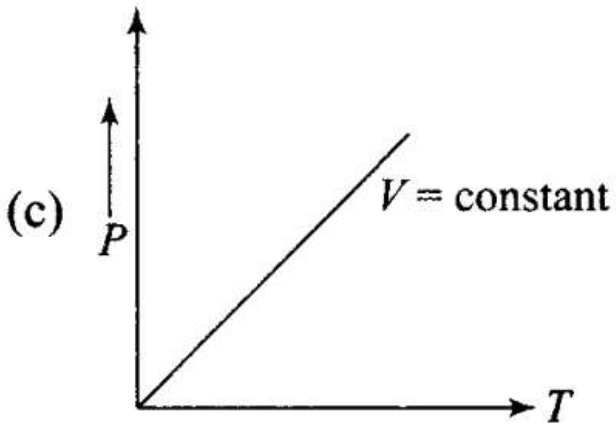
Kinetic energy per mole of gas depends only upon the temperature of gas.
Kinetic energy per gram of gas depend upon the temperature as well molecular weight (or mass of one molecule) of the gas.

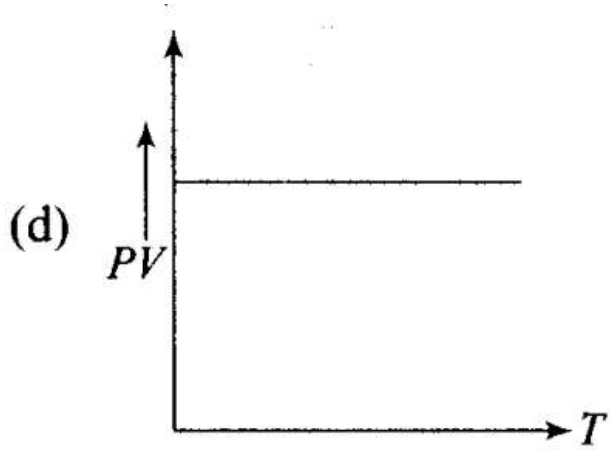
$$E_{gram} = \frac{3}{2}\frac{k}{m}T \qquad \therefore E_{gram} \propto \frac{T}{m}$$

From the above expressions it is clear that higher the temperature of the gas, more will be the average kinetic energy possessed by the gas molecules at T= 0, E = 0, i.e. at absolute zero the molecular motion stops.

Q12. Which of the following diagrams (figure) depicts ideal gas behaviour ?





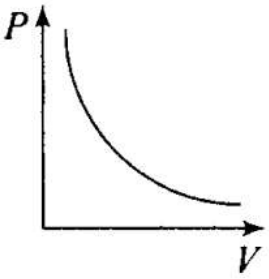


Sol. (a, c) For ideal gas behaviour,

$$PV = nRT \qquad \ldots(i)$$

(a) When pressure, P = constant
From (i) Volume $V \propto$ Temperature T
Graph of V versus T will be straight line.

(b) When T = constant
From (i) PV = constant
So, graph of P versus V will be a rectangular hyperbola. Hence this graph is wrong. The correct graph is shown below:



(c) When V = constant.
From (i) $P \propto T$
So, the graph is a straight line passing through the origin.

(d) From (i) $PV \propto T$

$$\therefore \Rightarrow \quad \frac{PV}{T} = \text{constant}$$

So, graph of PV versus T will be a straight line parallel to the temperature axis (x-axis).
i.e., slope of this graph will be zero.
So, (d) is not correct.

Q13. When an ideal gas is compressed adiabatically, its temperature rises the molecules on the average have more kinetic energy than before. The kinetic energy increases,
(a) because of collisions with moving parts of the wall only
(b) because of collisions with the entire wall
(c) because the molecules gets accelerated in their motion inside the volume
(d) because of redistribution of energy amongst the molecules
Sol:(a) Since the gas is ideal and the collisions of the molecules are elastic. When the molecules collides

with the moving parts of the wall, its kinetic energy increases. But the total kinetic energy of the system will remain conserved. When the gas is compressed adiabatically, the total work done on the gas increases, its internal energy which in turn increases the KE of gas molecules and hence, the collisions between molecules also increases.

Very Short Answer Type Questions

Q14. Calculate the number of atoms in 39.4 g gold. Molar mass of gold is 197 g mole^{-1}

Sol. Here, we know that,

Molar mass = Mass of Avogadro's number of atoms (Molecules)

$= 6.023 \times 10^{23}$ atoms.

According to the problem,

Mass of the gold, $m = 39.4$ g

Molar mass of the gold, $M = 197$ g mol^{-1}

Now, 197 g of gold contains 6.023×10^{23} atoms

So, 1 g of gold contains $\frac{6.023 \times 10^{23}}{197}$ atoms

$\therefore$ 39.4 g of gold contains $\frac{6.023 \times 10^{23} \times 39.4}{197}$ atoms $= 1.20 \times 10^{23}$ atoms

Q15. The volume of a given mass of a gas at 27°C, 1 atm is 100 cc. What will be its volume at 327°C?

Sol:

Key concept: Here the temperatures are given in Celsius. To apply ideal gas equation, we must convert the given temperature in kelvin. So, to convert them in kelvin we use the relation

$$\frac{C-0}{100}=\frac{K-273.15}{373.15-273.15}$$

Aecording to the problem, we have, $T_1 = 27°C$

Therefore, $\frac{27}{100}=\frac{T_1(K)-273.15}{373.15-273.15}$

$$T_1 \approx 300\ K$$

Similarly, $T_2 = 327°C \approx 600\ K$

$$V_1 = 100\ cm^3$$

If pressure of a given mass of the gas is kept constant, then according to isobaric process,

$$V \propto T$$

$$\Rightarrow \quad \frac{V}{T} = \text{constant} \qquad \begin{bmatrix} V = \text{Volume of gas} \\ T = \text{Temperature of gas} \end{bmatrix}$$

$$\Rightarrow \quad \frac{V_1}{T_1}=\frac{V_2}{T_2} \Rightarrow V_2 = V_1\left(\frac{T_2}{T_1}\right)$$

$$T_1 = 273 + 27 = 300\ K$$

$$T_2 = 273 + 327 = 600\ K$$

But $V_1 = 100$ cc

$$V_2 = V_1\left(\frac{600}{300}\right)$$

$$V_2 = 2V_1$$

$$\therefore \quad V_2 = 2 \times 100 = 200 \text{ cc}$$

Q16. The molecules of a given mass a gas have root **mean square speeds of 100 ms^{-1} at 27°C and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at 127°C and 2.0 atmospheric pressure?**

Sol: Key concept: Root Mean Square Speed: It is defined as the square root of mean of squares of the speed of different molecules

i.e., $$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + \cdots}{N}} = \sqrt{\overline{v^2}}$$

(i) From the expression of pressure $P = \frac{1}{3}\rho v_{rms}^2$

$$\Rightarrow \quad v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

where $\rho = \frac{\text{Mass of gas}}{V}$ = Density of the gas,

$M = \mu \times$ (mass of gas), $PV = \mu RT$, $R = kN_A$,

k = Boltzmann's constant, $m = \frac{M}{N_A}$ = mass of each molecule.

(ii) With rise in temperature, rms speed of gas molecules increases as $v_{rms} \propto \sqrt{T}$.

(iii) With increase in molecular weight rms speed of gas molecule decreases as $v_{rms} \propto \frac{1}{\sqrt{M}}$. e.g., rms speed of hydrogen molecules is four times that of oxygen molecules at the same temperature.

(iv) rms speed of gas molecules is of the order of km/s, e.g., at NTP for hydrogen gas $(v_{rms}) = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 273}{2 \times 10^3}} = 1840$ m/s

(v) rms speed of gas molecules is $\sqrt{\frac{3}{\gamma}}$ times that of speed of sound in gas, as $v_{rms} = \sqrt{\frac{3RT}{M}}$ and $v_s = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v_{rms} = \sqrt{\frac{3}{\gamma}} v_s$

(vi) rms speed of gas molecules does not depend on the pressure of gas (if temperature remains constant) because $P \propto \rho$ (Boyle's law)

If pressure is increased n times, the density will also increases by n times but v_{rms} remains constant.

According to the problem,

$(v_{rms})_1 = 100\ ms^{-1}$, $T_1 = 27°C = 300\ K$

$P_1 = 1\ atm$, $T_2 = 127°C = 400\ K$

$P_2 = 2\ atm$, $(v_{max})_2 = ?$

For a given mass of a gas rms velocity is given by the relation

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where, R is gas constant, T is temperature in kelvin, M is molar mass of gas.

Clearly, $v_{rms} \propto \sqrt{T}$

As R and M are constants,

$$\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$$

Therefore, $$\frac{100}{(v_{rms})_2} = \sqrt{\frac{300}{400}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \qquad v_{rms} \propto \sqrt{T} \Rightarrow \frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$$

Q17. Two molecules of a gas have speeds of $9 \times 10^{16}\ ms^{-1}$ and $1 \times 10^{6}\ ms^{-1}$ respectively. What is the root mean square speed of these molecules?

Sol: rms speed for w-molecules is defined as:

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \cdots + v_n^2}{n}} \quad [v_{rms} = \text{root mean square velocity}]$$

where $v_1, v_2, v_1, \ldots v_n$ are individual velocities of n-molecules of the gas.
For two molecules,

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

According to the problem, $v_1 = 9 \times 10^6$ m/s and $v_2 = 1 \times 10^6$ m/s

$$\therefore \quad v_{rms} = \sqrt{\frac{(9\times10^6)^2 + (1\times10^6)^2}{2}}$$

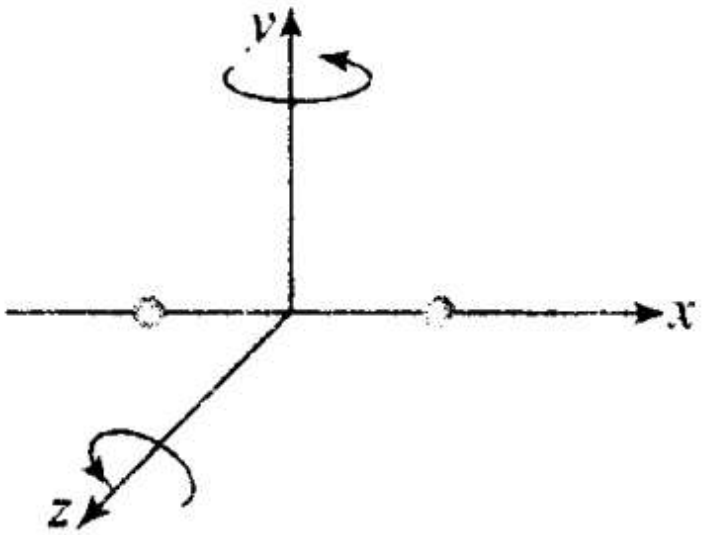
$$= \sqrt{\frac{81\times10^{12} + 1\times10^{12}}{2}}$$

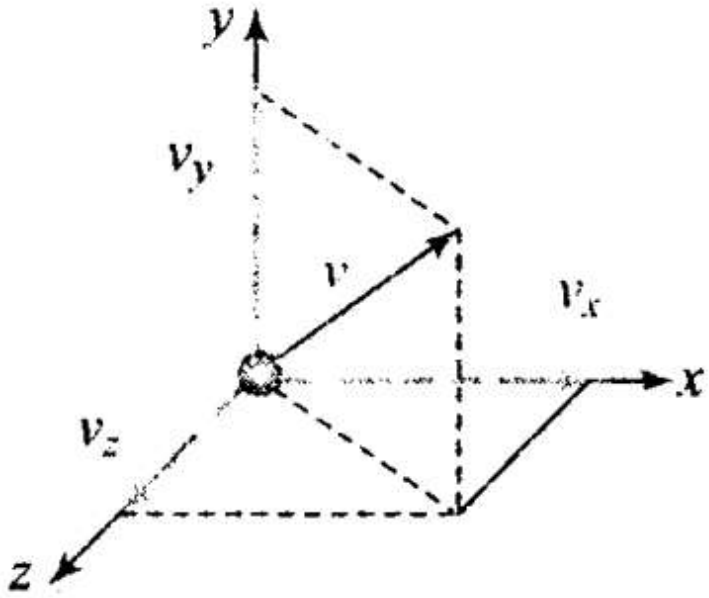
$$= 10^6\sqrt{\frac{81+1}{2}} = \sqrt{41}\times10^6 \text{ m s}^{-1}$$

Q18. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature T. Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes.)

Sol: Key concept: Degree of Freedom:
The term degree of freedom of a system refers to the possible independent motions, systems can have or
The total number of independent modes (ways) in which a system can possess energy is called the degree of freedom (f).
The independent motions can be translational, rotational or vibrational or any combination of these.
So the degree of freedom are of three types:
(i) Translational degree of freedom
(ii) Rotational degree of freedom
(iii) Vibrational degree of freedom
General expression for degree of freedom
f=3A- B; where A = Number of independent particles.

B = Number of independent restriction





Diatomic gas: Molecules of diatomic gas are made up of two atoms joined rigidly to one another through a bond. This cannot only move bodily, but also rotate about one of the three co-ordinate axes. However its moment of inertia about the axis joining the two atoms is negligible compared to that about the other two axes.

Hence it can have only two rotational motion. Thus a diatomic molecule has 5 degree of freedom: 3 translational and 2 rotational.

Monoatomic gas: Molecules of

monoatomic gas can move in any direction in space so it can have three independent motions and hence 3 degrees of freedom (all translational).

According to the problem, $n_o = 2$ moles, $n_n = 4$ moles
Temperature of the gas $= T$
Since, oxygen is a diatomic gas. So, degree of freedom associated with the molecules of the oxygen is 5.

Energy (total internal) per mole of the gas $= \frac{5}{2}RT$

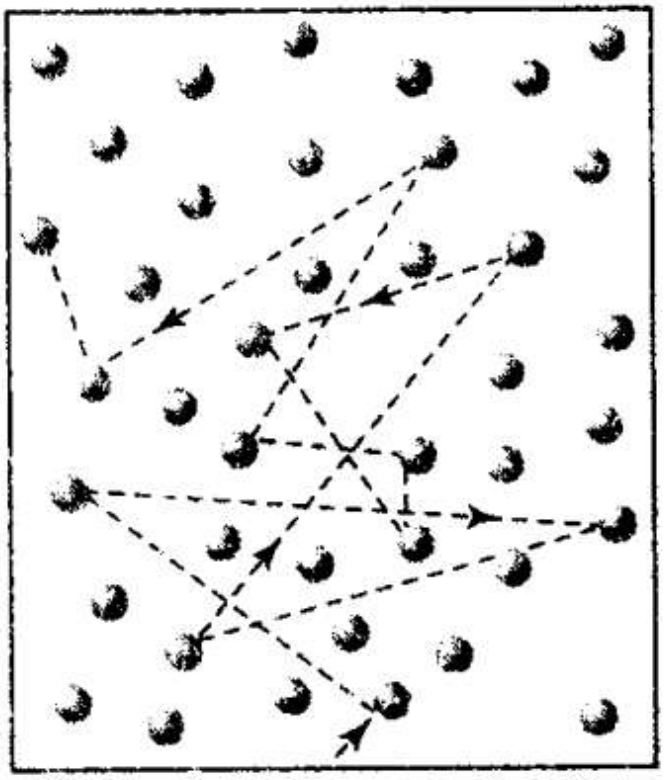
$$\begin{bmatrix} R = \text{universal gas constant} \\ T = \text{temperature} \end{bmatrix}$$

For 2 moles of the gas total internal energy $= 2 \times \frac{5}{2}RT = 5RT$ ...(i)

Neon (Ne) is a monoatomic gas having 3 degrees of freedom.
Energy per mole = 3/2RT
Hence, Energy = 4 x3/2 RT = 6RT ....(ii)
[Using Eqs. (i) and (ii)]
Total energy = 5RT = 6RT= 11RT

Q19. Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters 1 A and 2 A. The gases may be considered under identical conditions of temperature, pressure and volume.

Sol: Mean Free Path:
(1) The distance travelled by a gas molecule between two successive collisions is known as free path.
λ = Total distance travelled by a gas molecule between successive collisions / Total number of collisions
During two successive collisions, a molecule of a gas moves in a straight line with constant velocity.
Let $\lambda_1, \lambda_2, \lambda_3$ be the distance travelled by a gas molecule during n collisions respectively, then the mean free path of a gas molecule is given by



(2) $\lambda = \dfrac{1}{\sqrt{2}\pi n d^2}$

where d = Diameter of the molecule,
n = Number of molecules per unit volume

According to the problem, $d_1 = 1$ Å; $d_2 = 2$ Å
Since, mean free path,

$$\lambda \propto \frac{1}{d^2}$$

$$\Rightarrow \qquad \text{So, } \frac{\lambda_1}{\lambda_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$$

Hence, $\lambda_1 : \lambda_1 = 4 : 1$

Neon (Ne) is a monoatomic gas having 3 degrees of freedom.

$$\therefore \qquad \text{Energy per mole} = \frac{3}{2}RT$$

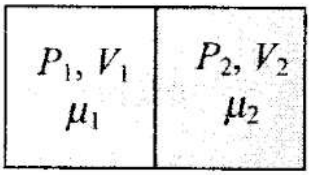
$$\text{Hence, Energy} = 4 \times \frac{3}{2}RT = 6RT \qquad \ldots(ii)$$

[Using Eqs. (i) and (ii)]

Total energy = $5RT = 6RT = 11RT$

Short Answer Type Questions

20. The container shown in figure has two chambers, separated by a partition, of volumes $V_1 = 2.0$ L and $V_2 = 3.0$ L. The chambers contain $\mu_1 = 4.0$ and $\mu_2 = 5.0$ mole of a gas at pressures $P_1 = 1.00$ atm and $P_2 = 2.00$ atm. Calculate the pressure after the partition is removed and the mixture attains equilibrium.



Sol. As shown in the diagram, container is divided in two chambers.

Volume of first chamber $V_1 = 2.0$ L,

Volume of second chamber $V_2 = 3.0$ L

$\mu_1 = 4.0$ mol, $\mu_2 = 5.0$ mol

$P_1 = 1.00$ atm, $P_2 = 2.00$ atm

According to the equation of ideal gas,

For chamber 1 $P_1V_1 = \mu_1RT_1$

For chamber 2 $P_2V_2 = \mu_2RT_2$

When the partition is removed, the gases get mixed without any loss of energy. The mixture now attains a common equilibrium pressure and the total volume of the system is sum of the volume of individual chambers V1 and V2. Let P be the pressure after the partition is removed.

So, $\mu = \mu_1 + \mu_2, V = V_1 + V_2$

From kinetic theory of gases,

For 1 mole $PV = \frac{2}{3}E$ $[E =$ translational kinetic energy$]$

For μ_1 moles, $P_1V_1 = \mu_1\left(\frac{2}{3}E_1\right)$

For μ_2 moles, $P_2V_2 = \mu_2\left(\frac{2}{3}E_2\right)$

Total energy of the gas in both the chambers $= (\mu_1E_1 + \mu_2E_2) = \frac{3}{2}(P_1V_1 + P_2V_2)$

From the above relation, $PV = \frac{2}{3}E_{\text{total}} = \frac{2}{3}\mu E_{\text{per mole}}$

$$P(V_1 + V_2) = \frac{2}{3} \times \frac{3}{2}(P_1V_1 + P_2V_2)$$

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$$

$$= \left(\frac{1.00 \times 2.0 + 2.00 \times 3.0}{2.0 + 3.0}\right)\text{atm}$$

$$= \frac{8.0}{5.0} = 1.60 \text{ atm}$$

Q21. A gas mixture consists of molecules of A, B and C with masses $m_A > m_B > m_C$. Rank the three types of molecules in decreasing order of (a) average KE, (b) rms speeds.

Sol. We know that the average KE of translation per molecule is

$$\text{KE} = \frac{3}{2} k_B T \qquad \ldots(\text{i})$$

Now as, $v_{\text{rms}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}}$

$$= \sqrt{\frac{3RT}{mN}} = \sqrt{\frac{3kT}{m}}$$

where, M = molar mass of the gas

m = mass of each molecular of the gas,

R = gas constant

Clearly, $v_{\text{rms}} \propto \sqrt{\frac{1}{m}}$

(a) From above equation (i) $\text{KE} \propto \sqrt{T}$

which remains same for all the three types of molecules, as conditions of temperature and pressure are the same.

(b) As k = Boltzmann constant

T = absolute temperature (same for all)

But $m_A > m_B > m_C$

$(v_{\text{rms}})_A < (v_{\text{rms}})_B < (v_{\text{rms}})_C$

$\therefore$ or $(v_{\text{rms}})_C > (v_{\text{rms}})_B > (v_{\text{rms}})A$

Q22. We have 0.5 g of hydrogen gas in a cubic chamber of size 3 cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100 atm. Is one justified in assuming the ideal gas law, in the final state? (Hydrogen molecules can be consider as spheres of radius 1 A).

Sol. According to the problem,

$$V_i = (3 \times 10^{-2})^3 = 27 \times 10^{-6}\ \text{m}^3$$

$$P_i = 1\ \text{atm},\ P_f = 100\ \text{atm},\ T = \text{constant}$$

Mass of hydrogen, $m = 0.5$ g

Thus at a constant pressure,

$$PV = nRT$$

where, n, R and T are constants. So,

$$\frac{V_f}{V_i} = \frac{P_i}{P_f}$$

Thus, final volume $V_f = \dfrac{P_i V_i}{P_f} = \dfrac{1 \times 27 \times 10^{-6}}{100}$

$$= 2.7 \times 10^{-7}\ \text{m}^3$$

Number of molecules in 0.5 g of H_2

$$N = \frac{N_A}{M} m$$

Here, $N_A = 6.022 \times 10^{23}\ \text{mol}^{-1}$

$M = 2\ \text{g mol}^{-1}$

$$\therefore \quad N = \frac{6.022 \times 10^{23}}{2} \times 0.5 = 1.5 \times 10^{23}$$

Radius of a hydrogen molecule = 1 Å = 10^{-10} m

Volume of a hydrogen molecule = $(10^{-10})^3 = 10^{-30}\ \text{m}^3$

Molecular volume = $(1.5 \times 10^{23}) \times 10^{-30}$

$$= 1.5 \times 10^{-7}\ \text{m}^3$$

We get V_f is of same order as the molecular volume. As the molecules lie very close to each other, so intermolecular forces cannot be ignored, i.e. gas cannot be treated as an ideal gas. Therefore, one is not justified in assuming the ideal gas law.

Q23. When air is pumped into a cycle tyre the volume and pressure of the air in the tyre both are increased. What about Boyle's law in this case?

Sol: Here, according to the question, when air is pumped, more molecules are pumped and Boyle's law is stated for situation where, mass of molecules remains constant.

$$PV = P\left(\frac{m}{\rho}\right) = \text{constant}$$

$$\Rightarrow \quad \frac{P}{\rho} = \text{constant} \quad \text{or} \quad \frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$$

$$\left(\text{As volume} = \frac{m}{\rho(\text{Density of the gas})} \text{ and } m = \text{constant}\right)$$

In this case, when air is pumped into a cycle tyre, mass of air in it increases as the number of air molecules keep increasing. Hence, this is a case of variable mass, Boyle's law (and even Charle's law) is only applicable in situations, where mass of gas molecules remains fixed. Hence, Boyle's law is not applicable in this case.

Q24. A balloon has 5.0 mole of helium at 7°C. Calculate
(a) the number of atoms of helium in the balloon.
(b) the total internal energy of the system.

Sol. Average KE per molecule $= \frac{3}{2}kT$

According to the problem, number of moles of helium, $n = 5$ gram mole

$$T = 7°C = 7 + 273 = 280 \text{ K}$$

(a) Hence, number of atoms (He is monoatomic)

Number of He atom, $N = n N_A$

$$= 5 \times 6.023 \times 10^{23}$$

$$= 30.015 \times 10^{23}$$

(b) As, KE $= \frac{3}{2}k_B T$

Here, k_B = Boltzmann constant.

$$\therefore \text{ Total internal energy } = \left(\frac{3}{2}kT\right) \times N$$

$$= \frac{3}{2} \times (1.38 \times 10^{-23}) \times 280 \times 30.115 \times 10^{23}$$

$$= 1.74 \times 10^4 \text{ J}$$

Important point: The above degrees of freedom are shown at room temperature. Further at high temperature, in case of diatomic or polyatomic molecules, the atoms with in the molecule may also vibrate with respect to each other. In such cases, the molecule will have an additional degrees of freedom, due to vibrational motion.

An object which vibrates in one dimension has two additional degrees of freedom. One for the potential energy and one for the kinetic energy of vibration. Helium is a mono atomic gas and It has only 3 degrees of freedom. But after addition its degree of freedom will be 5.

Q25. Calculate the number of degrees of freedom of molecules of hydrogen in 1 cc of hydrogen gas at NTP.

Sol:

Key concept: Total number of degrees of freedom in a thermodynamical system = Number of degrees of

freedom associated per molecule x number of molecules.

At NTP, Volume occupied by 6.023×10^{23} molecules of gas = 22400 cc

$$\therefore \text{ Number of molecules in 1 cc of hydrogen } = \frac{6.023 \times 10^{23}}{22400} = 2.688 \times 10^{19}$$

H_2 is a diatomic gas, having a total of 5 degrees of freedom.
(3 translational + 2 rotational)
∴ Total degrees of freedom possessed by all the molecules

$$= 5 \times 2.688 \times 10^{19} = 1.344 \times 10^{20}$$

Q26. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature.

Sol: Since, the container is suddenly stopped which is initially moving with velocity v_0, there is no time for exchange of heat in the process. Then total KE of the container is transferred to gas molecules in the form of translational KE, thereby increasing the absolute temperature.

Let n be the no. of moles of the monoatomic gas in the container. Since molar mass of the gas is m.

Total mass of the container, M = mn

KE of molecules due to velocity v_0,

KE = 1/2(mn) v_0^2

Final KE of gas = 0

Change in kinetic energy, $\Delta K = \frac{1}{2}(nm)v^2$

If ΔT = change in absolute temperature,

Then the internal energy of the gas

$$\Delta U = nC_v\Delta T = n\left(\frac{3}{2}R\right)\Delta T \qquad \text{...(ii)}$$

According to conservation of mechanical energy, we get

$$\Delta K = \Delta U$$

By equating Eqs. (i) and (ii),we get

$$\Rightarrow \qquad \frac{1}{2}(mn)v_0^2 = n\frac{3}{2}R(\Delta T)$$

$$(mn)v_0^2 = n3R(\Delta T)$$

$$\Rightarrow \qquad \Delta T = \frac{(mn)v_0^2}{3nR} = \frac{mv_0^2}{3R}$$