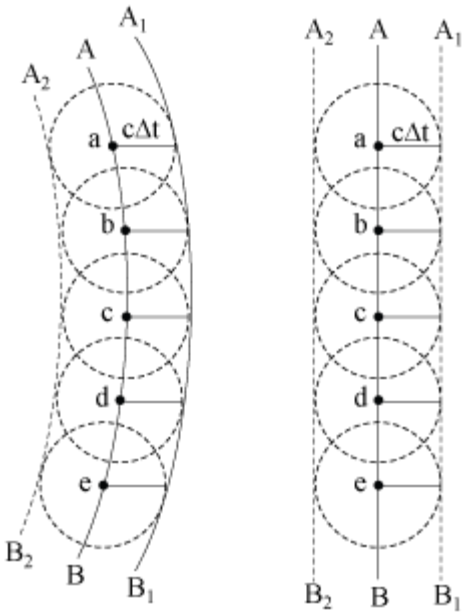
Wave Optics

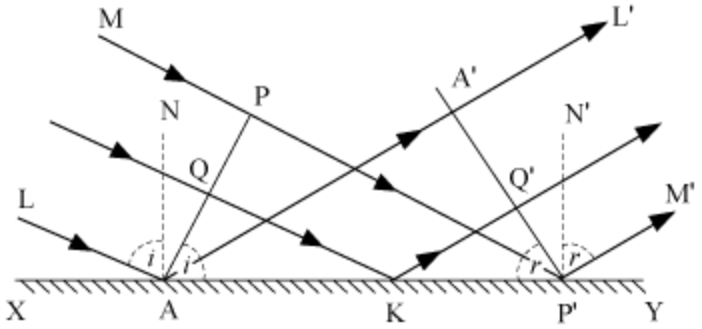
Huygens' Principle



Huygens' Principle is based on the following assumptions:

- Each point on the primary wavefront acts as a source of secondary wavelets, sending out disturbance in all directions in a similar manner as the original source of light does.
- The new position of the wavefront at any instant (called secondary wave front) is the envelope of the secondary wavelets at that instant.

Laws of Reflection on Wave Theory



- Consider any point Q on the incident wavefront PA.
- When the disturbance from P on incident wavefront reaches point P', the disturbance from point Q reaches Q'.

- If *c* is velocity of light, then time taken by light to go from point Q to Q' (via point K) is given by,

$$t = \frac{QK}{c} + \frac{KQ'}{c} \qquad \ldots(i)$$

- In right-angled ΔAQK,

  ∠QAK = *i*

  ∴ QK = AK sin *i*

- In right-angled $\Delta KQ'P'$,

$$\angle Q'P'K = r$$
$$\therefore KQ' = KP' \sin r$$

  Substituting these values in equation (1),

$$t = \frac{AK \sin i}{c} + \frac{KP' \sin r}{c}$$
$$t = \frac{AK \sin i + (AP' - AK) \sin r}{c} \left(\because KP' = AP' - AK\right)$$
$$t = \frac{AP' \sin r + AK(\sin i - \sin r)}{c} \qquad \ldots(ii)$$

  The rays from different points on incident wavefront will take the same time to reach the corresponding points on the reflected wavefront, if '*t*' given by equation (ii) is independent of AK.

  ∴ AK (sin *i* − sin *r*) = 0

  sin *i* − sin *r* = 0

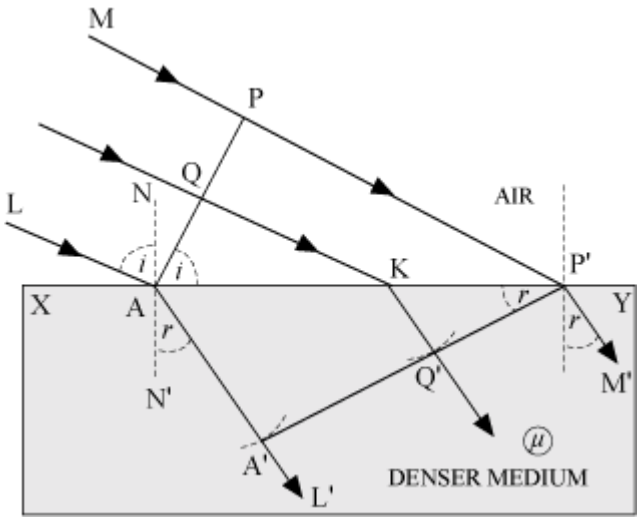
  sin *i* = sin *r*

  i = *r*

  i.e., the angle of incidence is equal to the angle of reflection.

  Also, the incident ray (LA or MP'), reflected ray ($AA'L'$ or $P'M'$), and the normal (AN) − all lie in the same plane.

Refraction On The Basis Of Wave Theory



- Consider any point Q on the incident wavefront.
- Suppose when disturbance from point P on incident wavefront reaches point P' on the refracted wavefront, the disturbance from point Q reaches Q' on the refracting surface XY.
- Since $P'A'$ represents the refracted wavefront, the time taken by light to travel from a point on incident wavefront to the corresponding point on refracted wavefront should always be the same. Now, time taken by light to go from Q to Q' will be

$$t = \frac{QK}{c} + \frac{KQ'}{v} \qquad \ldots(i)$$

- In right-angled ΔAQK, ∠QAK = i

  ∴ QK = AK sin i ... (ii)

- In right-angled $\Delta P'Q'K$,

  $\angle Q'P'K = r$

  $KQ' = KP' \sin r$ ... (iii)

- Substituting (ii) and (iii) in equation (i),

$$t = \frac{AK \sin i}{c} + \frac{KP' \sin r}{v}$$

$$\text{Or, } t = \frac{AK \sin i}{c} + \frac{(AP' - AK)\sin r}{v} \left(\because KP' = AP' - AK\right)$$

$$\text{Or, } t = \frac{AP'}{c} \sin r + AK\left(\frac{\sin i}{c} - \frac{\sin r}{v}\right) \qquad \ldots(iv)$$

- The rays from different points on the incident wavefront will take the same time to reach the corresponding points on the refracted wavefront i.e., *t* given by equation (iv) is independent of AK. It will happen so, if

$$\frac{\sin i}{c} - \frac{\sin r}{v} = 0$$

$$\frac{\sin i}{\sin r} = \frac{c}{v}$$

$$\text{However, } \frac{c}{v} = \mu$$

$$\therefore \mu = \frac{\sin i}{\sin r}$$

This is the Snell's law for refraction of light.

Doppler Effect & Coherent and Incoherent Addition of Waves

The Doppler Effect

- We should be careful in constructing the wavefronts, if the source (or observer) is moving.
- If the source moves away from the observer, then wavefronts have to travel a greater distance to reach the observer and hence, will take a longer time.
- Thus, when the source moves away from the observer, the frequency as measured by the source will be smaller. This is known as the Doppler effect.
- The increase in wavelength due to Doppler effect is red shift since a wavelength in the middle of the visible region of the spectrum moves towards the red end of the spectrum.
- When waves received from a source move towards the observer, there is an apparent decrease in wavelength. This is referred to as blue shift.
- The fractional change in frequency $\Delta\nu/\nu$ is given by $\frac{-v_{radial}}{c}$.

$$\frac{\Delta \nu}{\nu} = -\frac{\nu_{radial}}{c}$$

Where, ν_{radial} is component of the source velocity along the line joining the observer to the source relative to the observer

Coherent and Incoherent Addition of Waves

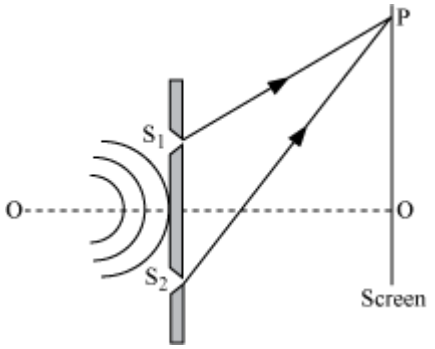
- Coherent sources: Two sources of light emitting light waves of same frequency or wavelength and of a stable phase difference

- Principle of superposition of light waves: When two or more wave trains of light travelling in a medium superpose upon each other, the resultant displacement at any instant is equal to the vector sum of the displacements due to individual waves.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3$, … be the displacement due to different waves, then the resultant displacement is given by

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots$$

- Conditions for constructive and destructive interference:



Let the displacement of the waves from the sources S_1 and S_2 at point P on the screen at any time 't' be given by,

$y_1 = a_1 \sin \omega t$

and

$y_2 = a_2 \sin (\omega t + \Phi)$

Where, Φ is the constant phase difference between the two waves

By the superposition principle, the resultant displacement at point P is given by,

$y = y_1 + y_2$

$y = a_1 \sin \omega t + a_2 \sin (\omega t + \Phi)$

$= a_1 \sin \omega t + a_2 \sin \omega t \cos \Phi + a_2 \cos \omega t \sin \Phi$

$y = (a_1 + a_2 \cos \Phi) \sin \omega t + a_2 \sin \Phi \cos \omega t$ ...(i)

Let $a_1 + a_2 \cos\phi = A\cos\theta$ ...(ii)

$a_2 \sin\phi = A\sin\theta$ ...(iii)

Then, equation (i) becomes

$$y = A\cos\theta \sin\omega t + A\sin\theta\cos\omega t$$

$$y = A\sin(\omega t + \theta)$$

Squaring and adding both sides of the equations (ii) and (iii), we obtain

$$A^2\cos^2\theta + A^2\sin^2\theta = (a_1 + a_2\cos\phi)^2 + a_2^2\sin^2\phi$$

$$A^2 = a_1^2 + a_2^2(\cos^2\phi + \sin^2\phi) + 2a_1a_2\cos\phi$$

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$

The intensity of light is directly proportional to the square of amplitude of the wave. The intensity of light at point P on the screen is given by,

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi \quad \text{...(iv)}$$

Constructive interference → cos Φ is Maximum i.e., cos Φ = + 1

$\Phi = 0, 2\pi, 4\pi, \ldots$

$$\phi = 2n\pi, \text{ where } n = 0, 1, 2, \ldots \quad \text{...(v)}$$

The phase difference Φ between the two waves will be

$$\phi = \frac{2\pi}{\lambda}x \quad \text{(vi)}$$

Where, x is the path difference, using equation (v)

The condition for constructive interference, that is equation (vi), becomes

$$\frac{2\pi}{\lambda}x = 2n\pi$$

$x = n\lambda$, where $n = 0, 1, 2, 3, \ldots$

It is the condition for constructive interference between two light waves in terms of path difference between them.

Destructive interference → From equation (iv), it follows that the intensity of light at point P will be minimum, if

$$\cos\phi = -1$$

$\Phi = \pi, 3\pi, 5\pi, \ldots$

$\Phi = (2n + 1)\,\pi$, where $n = 0, 1, 2, \ldots$ (vii)

From equations (vi) and (vii), we have

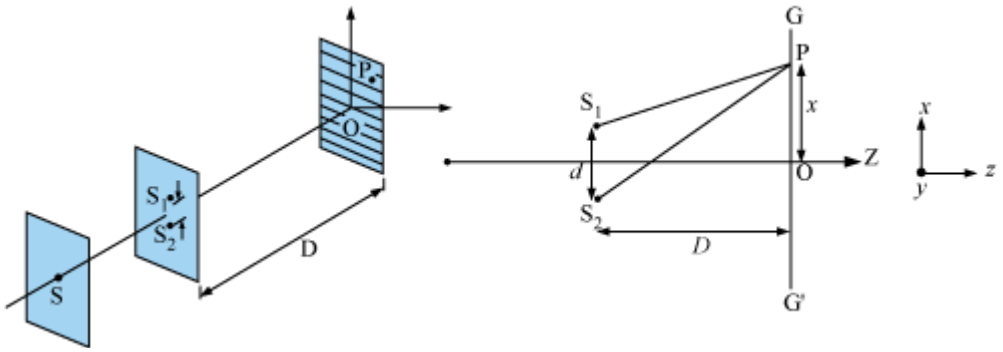
$$\frac{2\pi}{\lambda}x = (2n+1)\pi$$

$x = (2n+1)\frac{\lambda}{2}$, where $n = 0, 1, 2, \ldots$

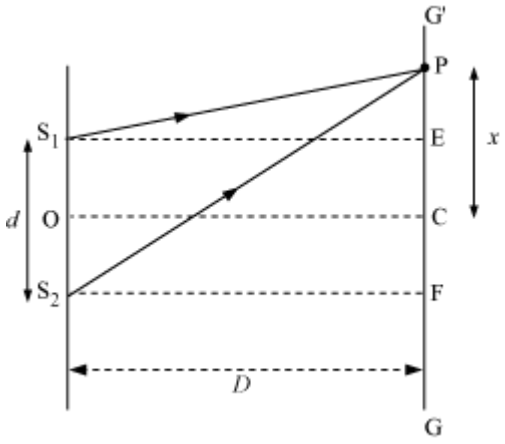
This is the condition for destructive interference in terms of phase difference and path difference.

Interference of Light Waves and Young's Experiment

- If we use two sodium lamps to illuminate two pinholes, then we will not observe any interference fringes.
- The British physicist Thomas Young made two pinholes S_1 and S_2 (very close to each other) on an opaque screen. These were illuminated by another pinhole that was, in turn, lit by a bright source. S_1 and S_2 behaved similar to two coherent sources because light waves coming out from S_1 and S_2 were derived from the same original source.



- Expression for Fringe Width in Young's Double-Slit Experiment



Let S_1 and S_2 be two slits separated by a distance *d.* GG' is the screen at a distance *D* from the slits $S_1$and S_2. Point C is equidistant from both the slits. The intensity of light will be maximum at this point because the path difference of the waves reaching this point will be zero.

At point P, the path difference between the rays coming from the slits S_1= S_2P - S_1P.

Now, S_1 S_2 = *d*, EF = *d*, and S_2F= *D.*

∴In ΔS_2PF,

$$S_2P = \left[S_2F^2 + PF^2\right]^{\frac{1}{2}}$$

$$S_2P = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right]^{\frac{1}{2}}$$

$$= D\left[1+\frac{\left(x+\frac{d}{2}\right)^2}{D^2}\right]^{\frac{1}{2}}$$

Similarly, in ΔS_1PE,

$$S_1P = D\left[1+\frac{\left(x-\frac{d}{2}\right)^2}{D^2}\right]^{\frac{1}{2}}$$

$$\therefore S_2P - S_1P = D\left[1+\frac{1}{2}\frac{\left(x+\frac{d}{2}\right)^2}{D^2}\right] - D\left[1+\frac{1}{2}\frac{\left(x-\frac{d}{2}\right)^2}{D^2}\right]$$

On expanding it binomially, we get:

$$S_2P - S_1P = \frac{1}{2D}\left[4x\frac{d}{2}\right] = \frac{xd}{D}$$

For bright fringes (constructive interference), the path difference is an integral multiple of wavelengths, i.e. path difference is $n\lambda$.

$$\therefore n\lambda = \frac{xd}{D}$$

$x = \frac{n\lambda D}{d}$, where n = 0, 1, 2, 3, 4, …

For n = 0, x_0 = 0

$$n = 1,\ x_1 = \frac{\lambda D}{d}$$

$$n = 2,\ x_2 = \frac{2\lambda D}{d}$$

$$n = 3,\ x_3 = \frac{3\lambda D}{d}$$

.

.

$$n = n,\ x_n = \frac{n\lambda D}{d}$$

Fringe width (β) → Separation between the centres of two consecutive bright fringes is called the width of a dark fringe.

$$\therefore \beta_1 = x_n - x_{n-1} = \frac{\lambda D}{d}$$

Similarly, for dark fringes,

$$x_n = (2n-1)\frac{\lambda}{2}\frac{D}{d}$$

For $n = 1,\ x_1 = \frac{\lambda D}{2d}$

For $n = 2,\ x_2 = \frac{3\lambda D}{2d}$

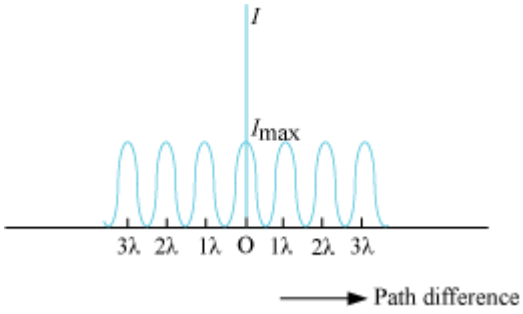
The separation between the centres of two consecutive dark interference fringes is the width of a bright fringe.

$$\therefore \boxed{\beta_2 = x_n - x_{n-1} = \frac{\lambda D}{d}}$$

$\therefore \beta_1 = \beta_2$

All the bright and dark fringes are of equal width, as $\beta_1 = \beta_2$.

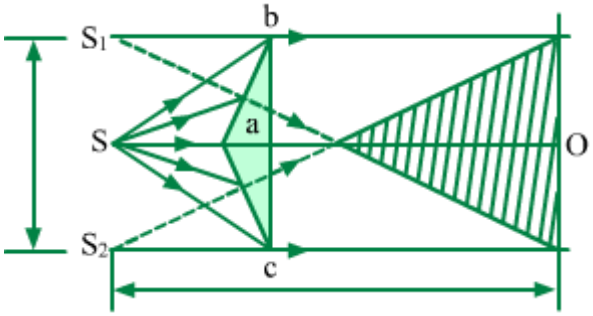
- Graph of intensity distribution in Young's Double-Slit Experiment:



Conditions For Producing Steady Interference Pattern

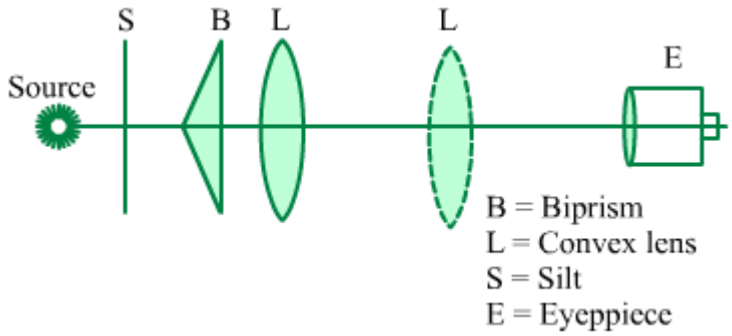
- The two sources of light must be coherent. The two sources of light waves are coherent if they emit light waves of same frequency, which are either in the same phase or with constant phase difference, which is possible if the two sources are derived from a single source.
- The sources of light must be monochromatic, i.e. they must emit light waves of only one wavelength.This is because due to the presence of lights of many wavelengths, the light waves form a constructive interference due to one wavelength and a destructive interference due to the other at the same point, leading to formation of a diffused and indistinct interference pattern.
- The two waves that are interfering must have the same state of polarisation.
- The two sources of light must emit light waves of same amplitude.
- The sources of light must be narrow. If the sources are not narrow, a number of overlapping interference patterns are observed due to light waves coming from many points of the source.
- The separation between the sources of light should be very small.
- The distance of the screen from the two sources of light should be large.
- The two interfering waves must travel in the same direction.

Measurement of Wavelength by Bi-prism Experiment



A bi-prism is made by joining two identical thin prisms of very small refracting angle (30' to 1^{o}). The angle of prism for the bi-prism is 179^{o}. The bi-prism is used to derive the two coherent sources from a single monochromatic source of light slit S.

Bi-prism Experiment



The slit, bi-prism, microscope and eye-piece are mounted on an optical bench, all set at almost the same height. The microscope eye-piece has a cross-wire and a Vernier scale for the measurement of the fringe width. The slit is illuminated by a sodium vapour lamp.

Keeping the slit vertical, the bi-prism is rotated slowly about the horizontal axis, such that the refracting edge becomes parallel to the slit.The interference fringes can be observed in the region where the two beams overlap.

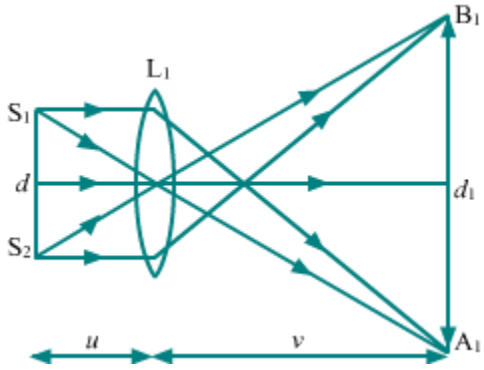
The wavelength of light can be determined by the measurement of the fringe width (β).

$\beta = \lambda D/d$

$\therefore \lambda = \beta d/D$

Measurements:

- Distance between the slit and the eye-piece, *D,* can be easily measured by the scale on the optical bench.
- The fringe width(β) can be measured with the help of a micrometer eyepiece.

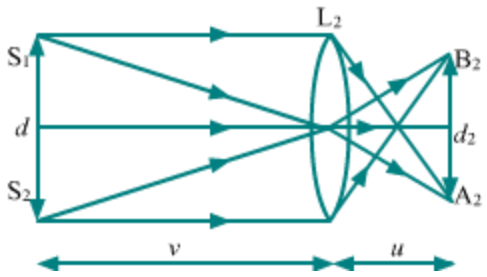


- The distance *d* between the two virtual sources S_1 and S_2 is measured using the conjugate foci method. For this, a convex lens of short wavelength is introduced between the eye-piece and the bi-prism. The eye-piece is moved back so that its distance from the slit becomes four times the focal length of the lens. The lens is moved towards the slit and its position is adjusted, so that the images of the virtual sources S_1 and S_2 are formed on the focal plane of the eye-piece.

If the distance between the images of the sources at the focal plane are found to be d_1,

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Image distance}}{\text{Object distance}}$$

$$\frac{d_1}{d} = \frac{v}{u} \quad \ldots. (1)$$



The lens is now moved towards the eye-piece, where two diminished images of S_1 and S_2 are formed on the focal plane. The distance between the images d_2 is measured by the micrometer. According to the principle of conjugate foci,

$$\frac{d_2}{d} = \frac{u}{v} \quad \ldots (2)$$

Multiplying (1) and (2), we get:

$$\frac{d_1 d_2}{d^2} = \frac{u}{v} \times \frac{v}{u} = 1$$

$$\Rightarrow d^2 = d_1 d_2$$

$$\Rightarrow d = \sqrt{d_1 d_2}$$

In this way, by calculating the values of *d, D* and β, the values of the wavelengths can be calculated.

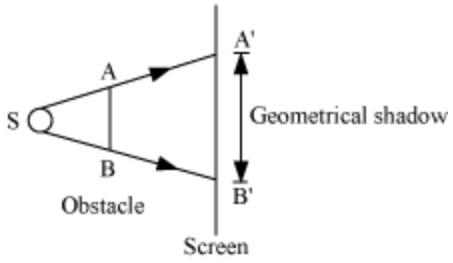
Diffraction

Types of Diffraction:

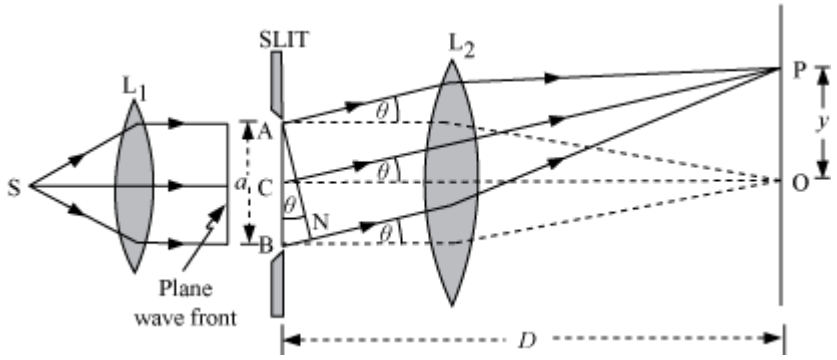
- **Fraunhofer Diffraction:**
- The source of light and the screen on which the diffraction pattern is observed are at infinite distances.
- Here, the plane wavefront is considered.
- A convex lens is used to obtain the diffraction pattern.
- **Fresnel Diffraction:**
- The source of light and the screen are at finite distances.
- Here, cylindrical or spherical wavefronts are considered.

Diffraction at a single slit (Fraunhofer diffraction)

The phenomenon of light bending around the sharp corners of an obstacle and spreading into the regions of the geometrical shadow is called diffraction.



- **Expression For Fringe Width**



Consider a parallel beam of light from a lens falling on a slit AB. As diffraction occurs, the pattern is focused on the screen XY with the help of lens L_2. We will obtain a diffraction pattern that is a central maximum at the centre O, flanked by a number of dark and bright fringes called secondary maxima and minima.

Central Maximum – Each point on the plane wave front AB sends out secondary wavelets in all directions. The waves from points equidistant from the centre C, lying on the upper and lower half, reach point O with zero path difference and hence, reinforce each other producing maximum intensity at point O.

- **Positions and Widths of Secondary Maxima and Minima**

Consider a point P on the screen at which wavelets travelling in a direction, and making angle θ with CO, are brought to focus by the lens. The wavelets from points A and B will have a path difference equal to BN.

From the right-angled ΔANB,

$BN = AB \sin\theta$

$BN = a \sin\theta$ ...(i)

Suppose $BN = \lambda$ and $\theta = \theta_1$.

Then, the above equation gives the following relation:

$\lambda = a \sin \theta_1$

$$\boxed{\sin \theta_1 = \frac{\lambda}{a}} \qquad \text{...(ii)}$$

Such a point on the screen will be the position of the first secondary minimum.

If BN = 2λ and $\theta = \theta_2$, then

$2\lambda = a \sin \theta_2$

$$\sin \theta_2 = \frac{2\lambda}{a} \qquad \text{...(iii)}$$

Such a point on the screen will be the position of the second secondary minimum.

In general, for n^{th} minimum at point P,

$$\sin \theta_n = \frac{n\lambda}{a} \qquad \text{...(iv)}$$

If y_n is the distance of the n^{th} minimum from the centre of the screen, then from right-angled ΔCOP,

$$\tan \theta_n = \frac{\text{OP}}{\text{CO}}$$

$$\tan \theta_n = \frac{y_n}{D} \qquad \text{...(v)}$$

In case θ_n is small, $\sin \theta_n \approx \tan \theta_n$

∴ From equations (iv) and (v), we get:

$$\frac{y_n}{D} = \frac{n\lambda}{a}$$

$$y_n = \frac{nD\lambda}{a}$$

Width of the secondary maximum,

$$\beta = y_n - y_{n-1} = \frac{nD\lambda}{a} - \frac{(n-1)D\lambda}{a}$$

$$\beta = \frac{D\lambda}{a} \qquad \text{...(vi)}$$

$\because \beta$ is independent of n, all the secondary maxima are of the same width β.

If $\mathrm{BN} = \frac{3\lambda}{2}$ and $\theta = \theta_1'$, then from equation (i), we get:

$$\sin\theta_1' = \frac{3\lambda}{2a}$$

Such a point on the screen will be the position of the first secondary maximum.

Corresponding to path difference,

$\mathrm{BN} = \frac{5\lambda}{2}$ and $\theta = \theta_2'$, the second secondary maximum is produced.

In general, for the n^{th} maximum at point P,

$$\sin\theta_n' = \frac{(2n+1)\lambda}{2a} \qquad \text{(vii)}$$

If y_n' is the distance of the n^{th} maximum from the centre of the screen, the angular position of the n^{th} maximum,

$$\tan\theta_n' = \frac{y_n'}{D} \qquad \text{(viii)}$$

In case θ_n' is small,

$$\sin\theta_n' \approx \tan\theta_n'$$

$$\therefore y_n' = \frac{(2n+1)D\lambda}{2a}$$

Width of the secondary minimum,
β' = y'n − y'n−1 =nDλa − (n−1)Dλaβ' = y'n - y'n-1 =nDλa - (n-1)Dλa

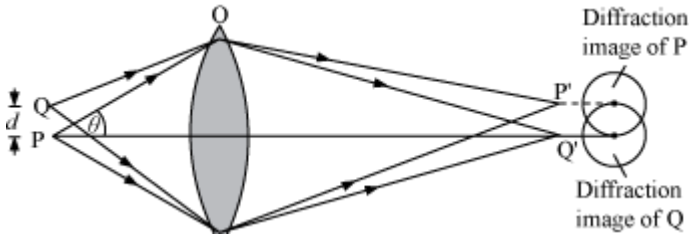
$$\boxed{\beta' = \frac{D\lambda}{a}} \qquad \text{(ix)}$$

Since β' is independent of *n*, all the secondary minima are of the same width β'.

Resolving Power of Optical Instruments

- **Resolving power of a microscope**:

  It is defined as the reciprocal of the least separation between two close objects, so that they appear just separated when seen through the microscope.



∴ Resolving power of a microscope,

$$\frac{1}{d} = \frac{2\mu \sin\theta}{1.22\lambda}$$

Here,

d → Least distance between the two objects, so that their diffraction images are just resolved

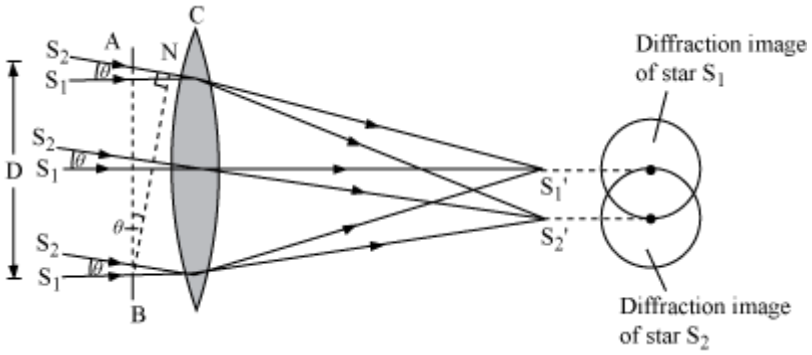
θ → Semi-vertical angle of the cone in which rays of light from an object enter the objective of the microscope

μ → Refractive index of the medium

λ → Wavelength of the light used to observe the objects

- **Resolving Power of a Telescope**

  Resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects, so that they appear just separated when seen through the telescope.



Resolving power of a telescope,

$$\frac{1}{\theta} = \frac{D}{1.22\lambda} \qquad \text{(i)}$$

Here,

θ → Angular separation between the two stars

λ → Wavelength of the light in which the two stars are observed

D → Diameter of the objective O of the telescope

Rayleigh's Criterion

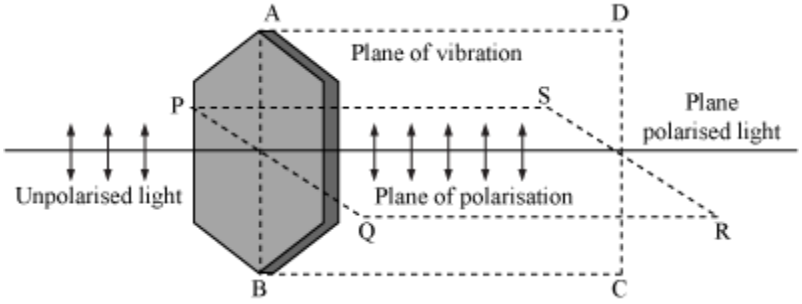
- According to Rayleigh's Criterion, two point objects are said be just resolved when the central maximum of the diffraction pattern of one lies on the first secondary minimum of the other.
- The two objects are said to be well-resolved, when the separation of the central maximum of the two objects is greater than the distance between the central maximum and first minimum of any of the two objects.
- If the separation between the central maximum of the two objects is less than the distance between the central maximum and first minimum of any of the two objects, then the objects cannot be seen distinctly and hence, are said to be unresolved.

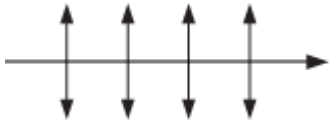
Difference between Interference and Diffraction

- Interference occurs due to superposition of two distinct waves coming from two coherent sources of light. The diffraction occurs as a result of the secondary wavelets coming from different parts of the same wavefront.
- In the pattern of the interference, all the bright fringes have same intensity. In a diffraction pattern, all the bright fringes are not of the same intensity.
- In an interference pattern, the dark fringe has zero or very small intensity, and the bright and dark fringes can easily be distinguished. In diffraction, all the dark fringes are not of zero intensity.
- In interference, the widths of all the fringes are almost same, whereas in diffraction, the fringes are of different widths.

Polarisation

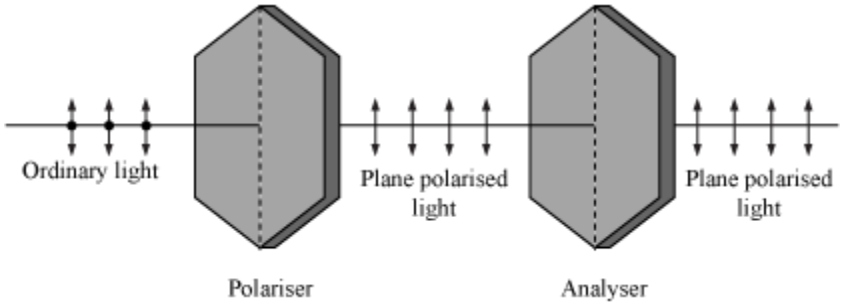
- The phenomenon due to which the vibrations of light are restricted to a particular plane is called polarisation of light.

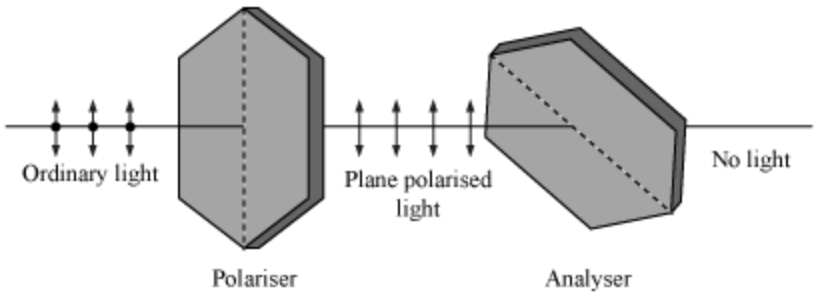


	
Plane polarised light with vibrations in the plane of the paper	Plane polarised light with vibrations in a plane perpendicular to the plane of the paper

- **To Detect Plane Polarised Light**

  The naked eye or the polariser cannot distinguish between unpolarised and plane polarised light. A crystal called analyser is used to analyse the nature of light.

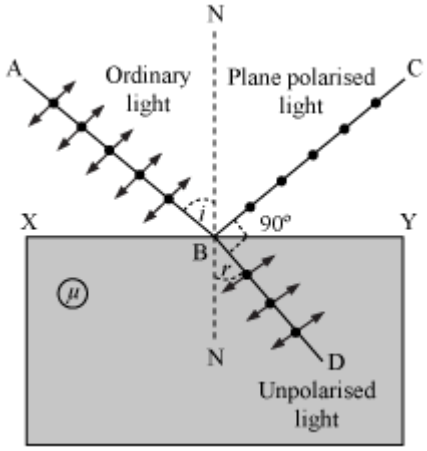




If the axes of the polariser and the analyser are parallel to each other, the intensity of light is found to remain unaffected.

The intensity of light becomes minimum when the axes of the analyser and polariser are perpendicular to each other.

- **Polarisation by Reflection**



An ordinary beam of light, on reflection from a transparent medium, becomes partially polarised. The degree of polarisation increases as the angle of incidence is increased. At a particular value of the angle of incidence, the reflected beam becomes completely polarised. This angle of incidence is called the polarising angle (p).

- **Brewster's Law**

When light is incident at the polarising angle at the interface of a refracting medium, the refractive index of the medium is equal to the tangent of the polarising angle.

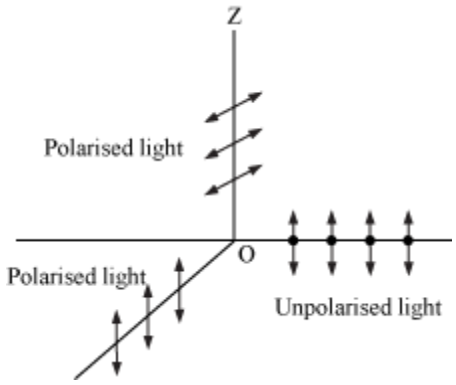
$$\mu = \tan p$$

Here,

μ → Refractive index of the refracting medium

$p \rightarrow$ Polarising angle

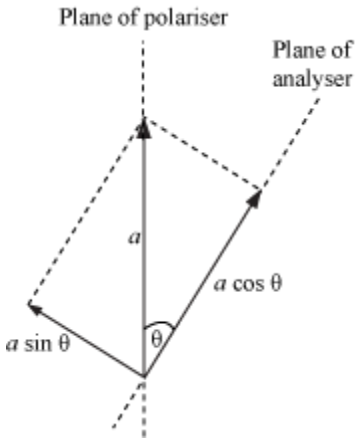
- **Polarisation by Scattering**



When a beam of light passes through a medium, it gets scattered from the particles constituting the medium, provided the size of the particles is of the order of the wavelength of the light. The scattered light, viewed in a direction perpendicular to the direction of the beam of light, is found to be plane polarised. It gives us a method to produce plane polarised light by scattering.

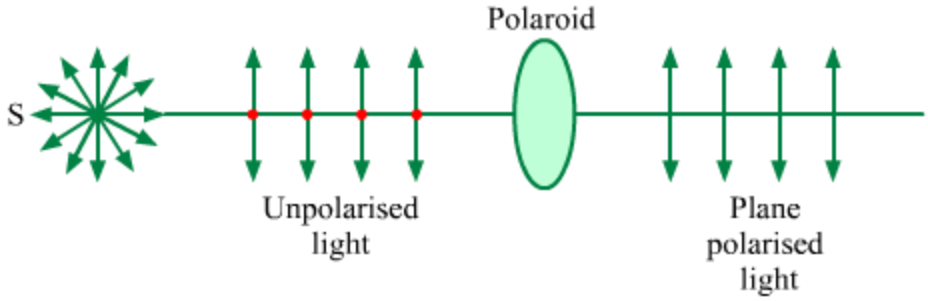
- **Law of Malus**

It states that when a completely plane polarised light beam is incident on an analyser, the intensity of the emergent light varies as the square of the cosine of the angle between the plane of transmission of the analyser and the polariser.

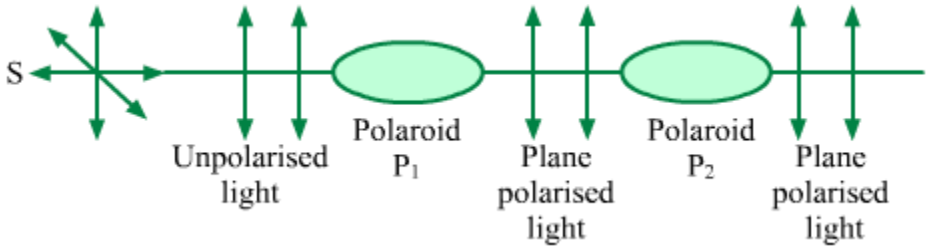


$$\boxed{I \propto \cos^2 \theta}$$

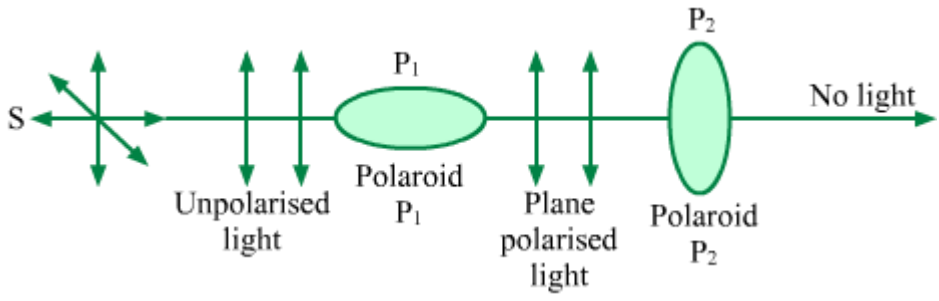
Polaroid:



- It is a large sheet of synthetic material packed with tiny crystals of a dichoric substance, oriented parallel to one another. They are capable of blocking one of the two planes of vibration of an electromagnetic wave.
- **Dichroism**: It is the property due to which a doubly-refracting crystal completely absorbs the ordinary rays, whose direction is parallel to the optic axis while passing through the crystal.
- If an unpolarised light wave is incident on a polaroid, the light wave will get linearly polarised with the electric vector oscillating along a direction perpendicular to the aligned molecules; this direction is known as the **pass-axis** of the polaroid.



When the unpolarised light from an ordinary source passes through a polaroid P_1, the intensity of the light received at the other end is reduced by half. On placing an identical piece of polaroid P_2 before P_1, such that both their axs are parallel, the light from the lamp is reduced in intensity on passing through P_2 alone.



When P_2 is rotated, the intensity of light received is gradually decreased, but when axes of P_1 and P_2 become perpendicular, the light coming from P_2 has nearly zero intensity.

Uses of Polaroid

The following are some uses of polaroids:

- Polaroids are used in three-dimensional movie cameras.
- Polaroids are used to produce and analyse plane polarised light.
- Polarised windows are used in airplanes to control the amount of light.
- Polaroid glasses are used in sunglasses to protect the eyes from sunlight.
- Polaroids are used in improving the colour contrast in old oil paintings.